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Why numerical simulations?

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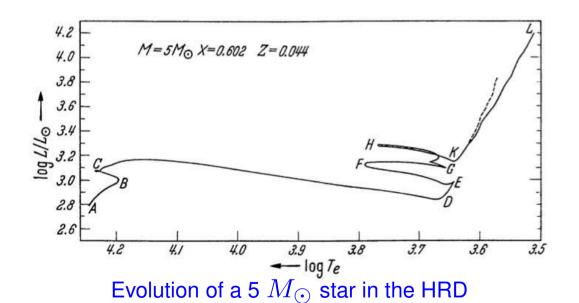
Part I: Aspects of computational astrophysics



§ 1 The role of computer simulations in astrophysics

Historical Perspective:

- 1950's and 1960's: Stellar evolution calculations (Martin Schwarzshild in the U.S. and Rudolph Kippenhahn in Göttingen, Germany). At that time *computers were viewed* as tools for the numerical integration rather than as a tool for experimentation.

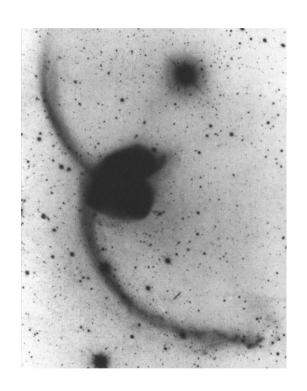


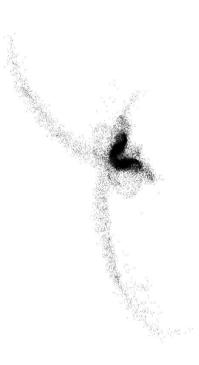
Kippenhahn et al. (1965, Zeitschrift für Astrophysik)



Rudolph Kippenhahn

- 1. The role of computer simulations in astrophysics (cont.)
- 1960's: N-body stellar dynamics simulations (e.g. tidal interaction of galaxies) and hydrodynamical systems (e.g. core collapse supernovae). Notion of *computational* astrophysics as experimental astronomy.





The antenna nebula NGC 4038/4039 evolved from a collision of two similarly sized spiral galaxies. *Left:* Observed present state. *Right:* Present state from a computer simulation of the complete collision (www.ifa.hawaii.edu/barnes).

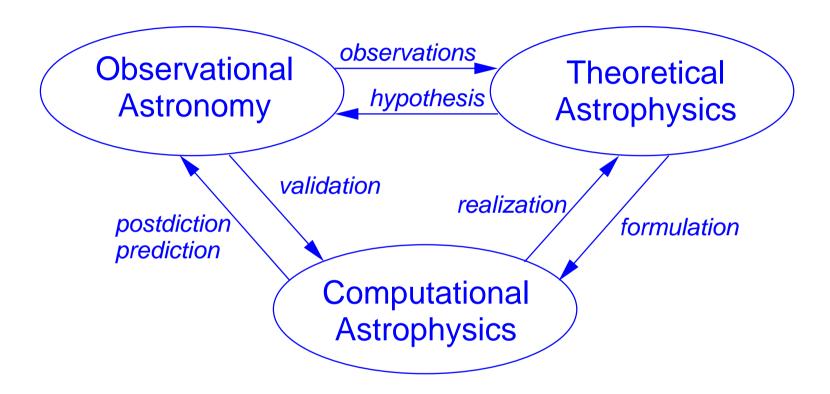
These simulations are generally *motivated by the question "What happens if?"* more so than "What is the solution to these equations?".

1. The role of computer simulations in astrophysics (cont.)

Computational astrophysics is the experimentation with astrophysical objects in a virtual (numerical) laboratory, comparable to the manipulation with real probes in classical physics experiments.

1. The role of computer simulations in astrophysics (cont.)

Role of Computational astrophysics:



Adapted from *M. Norman* (1997)

1. The role of computer simulations in astrophysics (cont.)

Simulations tend to model a *time dependent* physical system with some degree of *realism*. Usually, simulated systems have no simple closed form analytic solutions. Otherwise, one rather talks of *modeling*.

Realistic simulations produce observable quantities like intensity maps or polarimetric maps that look like corresponding actual observations, so called *virtual or synthetic observations*. Typically, a realistic solar simulation uses a *realistic equation of state* that takes ionization and the composition of the solar plasma into account and it carries out *radiation transfer with actual opacities* as occurring in the solar plasma.

References

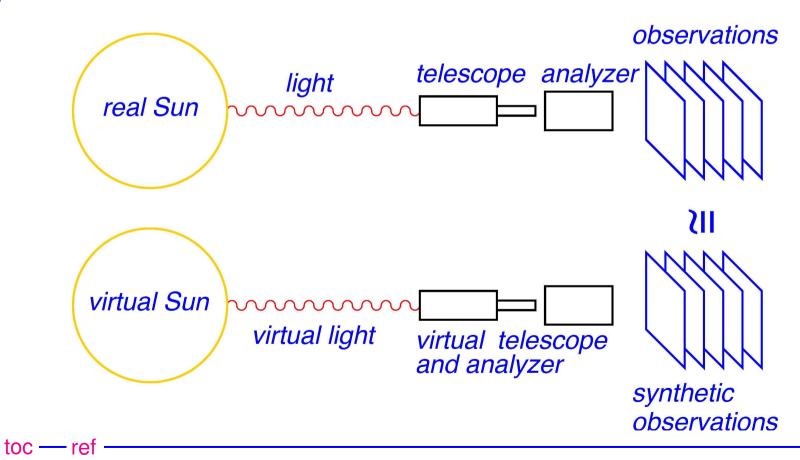
- Kippenhahn, R., Thomas, H.-C., and Weigert, A.: 1965, Sternetnwicklung IV. Zentrales Wasserstoff- und Heliumbrennen bei einem Stern von 5 Sonnenmassen, Zeitschrift für Astrophysik 61, 241-267
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Part II: MHD simulations: Case studies



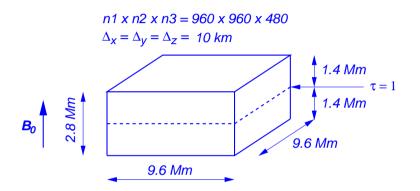
§ 2 Observations vs. simulations of the solar atmosphere

The solar atmosphere is in continuous turbulent motion. Light that reaches us from the Sun bears information on this dynamics and it's magnetic field. But this information is not easy to decipher. *Numerical simulations* help us to do this.



2. Observations vs. simulations of the solar atmosphere (cont.): nMBPs



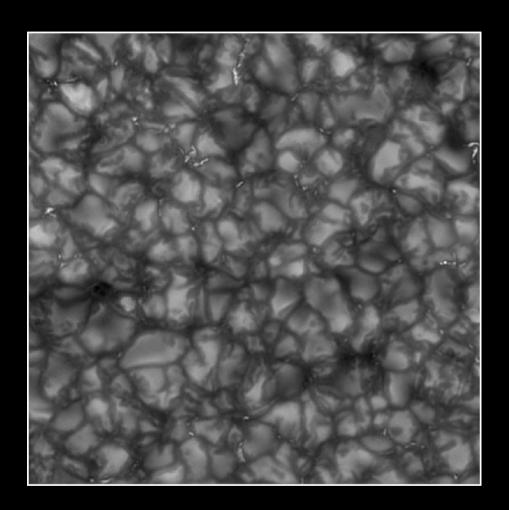


convection zone

convection zone base

Size of a typical three-dimensional computational domain in comparison with the size of the Sun.

2. Observations vs. simulations of the solar atmosphere (cont.)



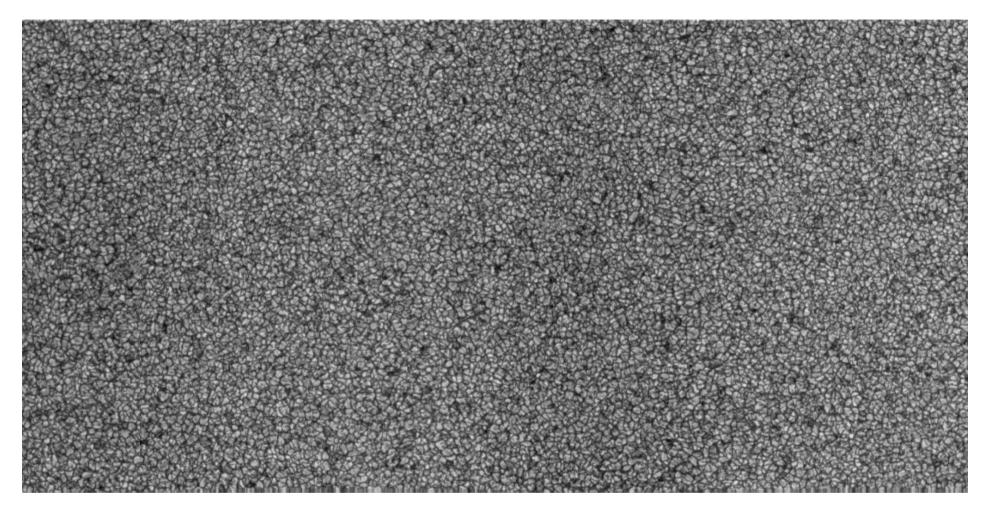
Observation of the solar surface

Numerical simulation

§ 3 Case study I: Interpretation of polarimetric signals

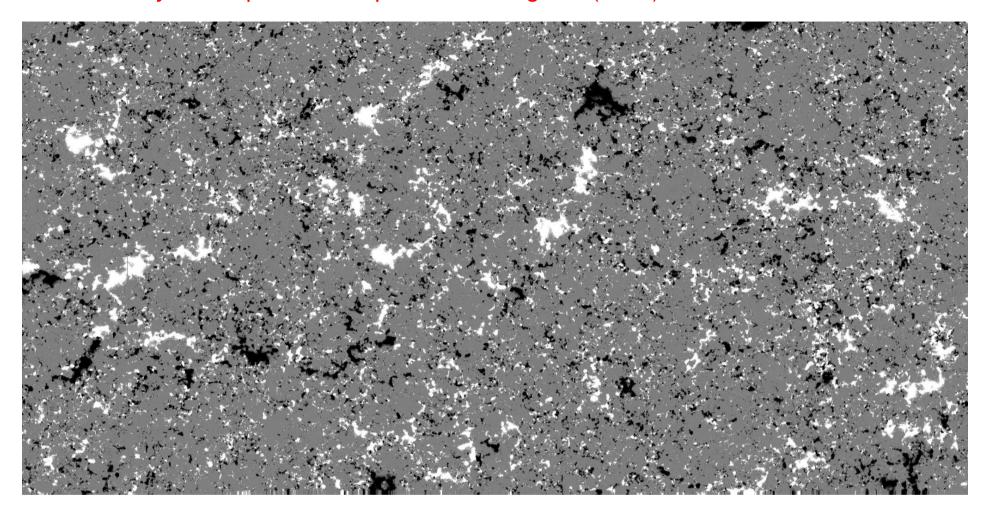
Case study I is a typical example of *post diction*: Something that was observed is *reproduced with simulations*, which helps us to better understand and interpret the observation.

3. Case study I: Interpretation of polarimetric signals (cont.)



Continuum intensity at 630 nm over a field of view of $302^{\prime\prime}\times162^{\prime\prime}$ recorded with Hinode/SOT/SP. 2048 slit positions. From *Lites et. al. 2008, ApJ 672, 1237*

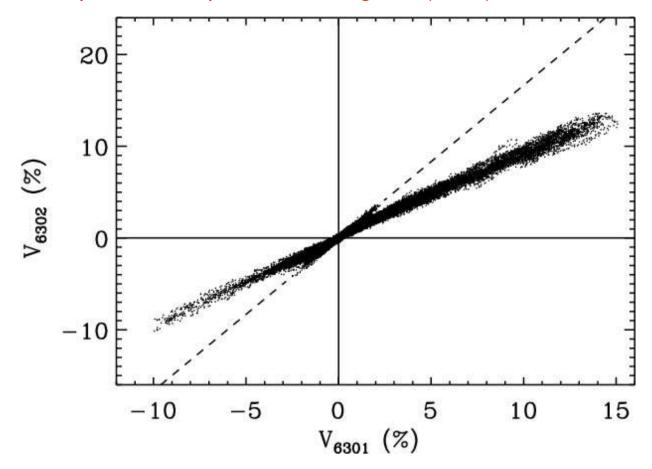
3. Case study I: Interpretation of polarimetric signals (cont.)



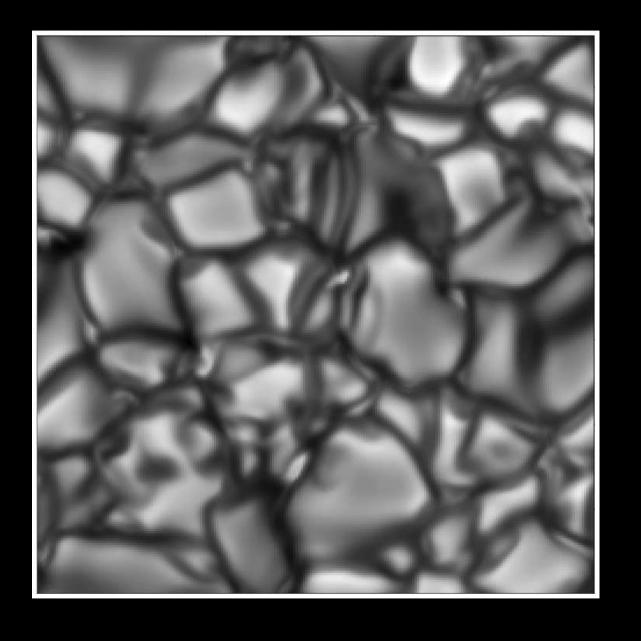
Apparent vertical magnetic flux density, $B_{\rm app}^{\rm L}$, of the quiet Sun over a field of view of $302'' \times 162''$. 2048 steps to 4.8 s. Maps of Fe I 630.15 and 630.25 nm.

From Lites et. al. 2008, ApJ 672, 1237

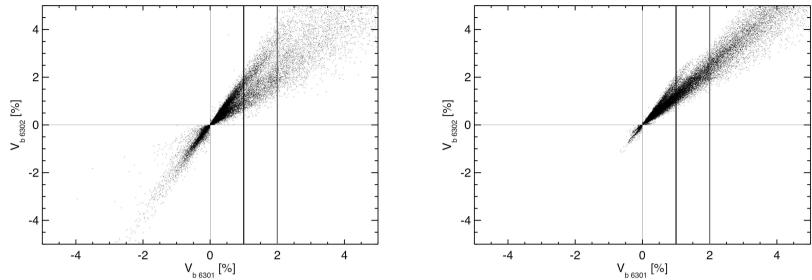
3. Case study I: Interpretation of polarimetric signals (cont.) observational data



Scatter plot of the blue lobe Stokes-V amplitudes of the 6302.5 Å line vs. the 6301.5 Å line as *observed with Hinode/SOT/SP*. The dashed line is the regression relation expected for weak magnetic fields. We identify two populations of points. From *Stenflo (2011) A&A 529 A42*. \Rightarrow Section at 1%

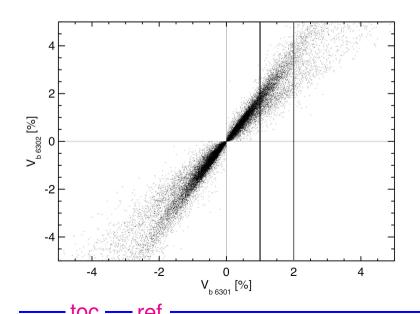


3. Case study I: Interpretation of polarimetric signals (cont.): simulation data



Scatter plot of the Stokes-V line ratio from the simulation. Left: full resolution;

Right: degraded with the SOT/SP point spread function. From Steiner & Rezaei (2012).



Scatter plot of the Stokes- ${\cal V}$ line ratio from a mixed polarity simulation.

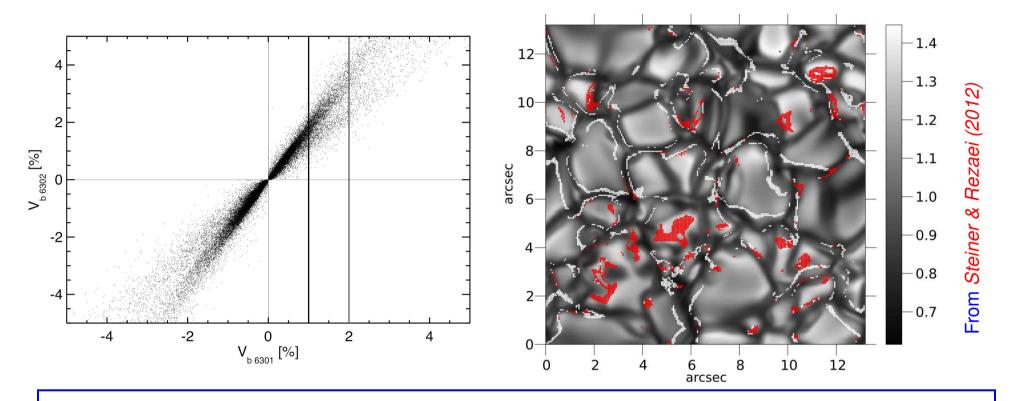
"It is nice to know that the computer understands the problem, but I would like to understand it too."

Attributed to E.P. Wigner

meaning:

"It is nice to know that our simulations reproduce the observations, but what can we learn from it?"

3. Case study I: Interpretation of polarimetric signals (cont.)



<u>Conclusion:</u> The two populations can be explained in terms of weak (hectogauss) magnetic fields. *Numerical simulations are indispensable for the correct interpretation.*



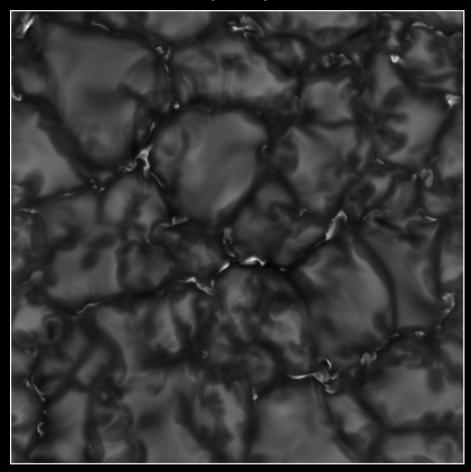
Maurizio Nannucci, Neon installation in the court of the Museo Novecento, Florence

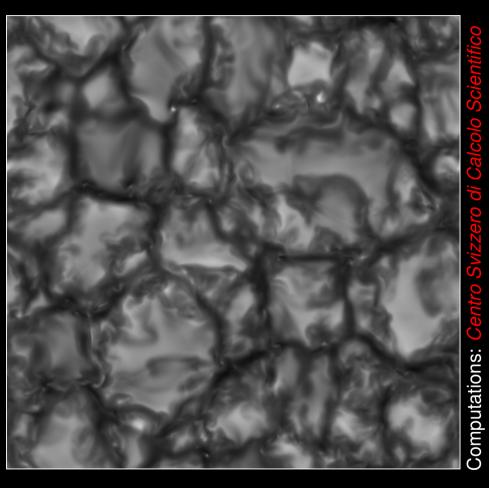
3. Case study I: Interpretation of polarimetric signals (cont.)

Case study I is a typical example of *post diction*: Something that was observed got *reproduced by simulations*, which helped us to better understand and interpret the observation.

The next paragraph treats an example of *prediction*.

Bolometric intensity maps



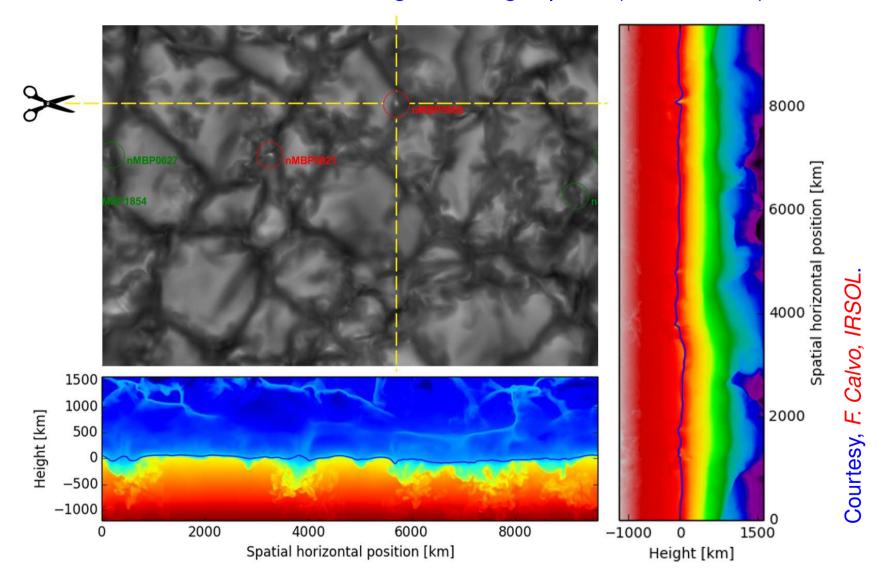


With magnetic fields: Magnetohydrodynamic simulation

Without magnetic fields: Hydrodynamic simulation

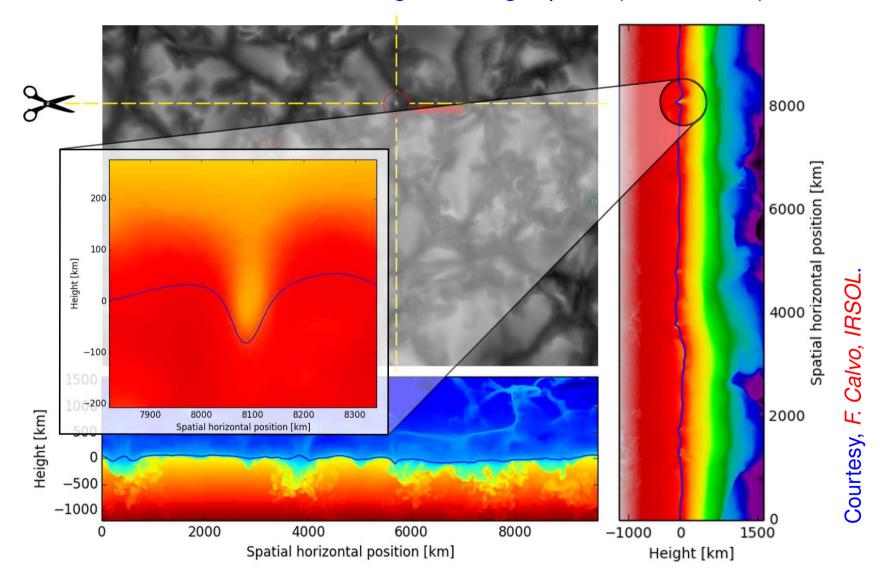
Courtesy, *F. Calvo*

Slices across a non-magnetic bright point (nMBP0868)

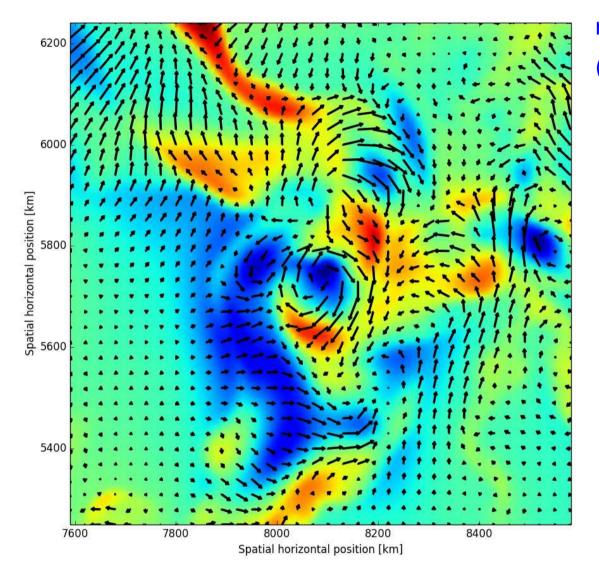


Emergent intensity I (top left), temperature T (bottom), density $\log(\rho)$ (right)

Slices across a non-magnetic bright point (nMBP0868)



Emergent intensity I (top left), temperature T (bottom), density $\log(\rho)$ (right)

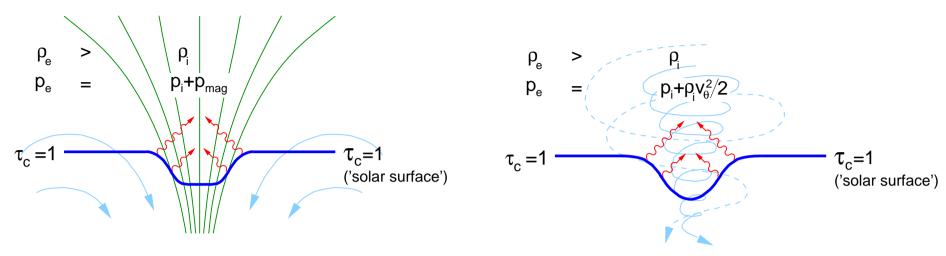


non-magnetic bright points (nMBPs) are locations with:

- $swirling\ motion$ (but $\approx 150\ [km]\ below\ au=1$ there are often swirls that do not produce nMBPs);
- low density (but a density deficiency alone does not warrant nMBP's);
- high intensity contrast (but a local intensity peak does not need to be a nMBP).

Density (blue: low, red: high) and velocity field in an horizontal plane, 150 [km] below $\langle \tau \rangle = 1$

From F. Calvo et al. 2016.



Magnetic flux sheet. Depression due to magnetic pressure.

Swirl. Depression due to centrifugal force.

In both cases is the 'hot wall effect' responsible for the enhanced radiation from the depression.

Reduction to an analytical toy model

Starting from the momentum equation

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \boldsymbol{\nabla}) \, \boldsymbol{v} + \frac{1}{\rho} \boldsymbol{\nabla} P + \boldsymbol{g} = 0,$$

we assume

- nMBPs are long-lived and stable so that the velocity field can be considered stationary;
- 2. They have cylindrical symmetry;
- 3. Their velocity field has a non-vanishing azimuthal component;
- 4. They extend in the vertical direction and their shape does not depend on depth.

Because of 2., the Euler momentum equation can be written in cylindrical coordinates.

Reduction to an analytical toy model

The advection term is then given by

$$(\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{v} = \left[(\boldsymbol{v}\cdot\boldsymbol{\nabla})v_r - \frac{v_\theta^2}{r}\right]\hat{\boldsymbol{r}} + \left[(\boldsymbol{v}\cdot\boldsymbol{\nabla})v_\theta + \frac{v_\theta v_r}{r}\right]\hat{\boldsymbol{\theta}} + (\boldsymbol{v}\cdot\boldsymbol{\nabla})v_z\hat{\boldsymbol{z}},$$

where the directional derivative is

$$\boldsymbol{v} \cdot \boldsymbol{\nabla} = v_r \partial_r + \frac{v_\theta}{r} \partial_\theta + v_z \partial_z$$
.

The simplest field satisfying the conditions 1–4 is $\mathbf{v} = v_{\theta}(r) \hat{\boldsymbol{\theta}}$. Inserting it into the Euler momentum equation and projecting it into the horizontal plane yields

$$\frac{\partial P}{\partial r} - \rho \frac{v_{\theta}^2}{r} = 0.$$

The pressure gradient is provided by the centripetal force.

Reduction to an analytical toy model

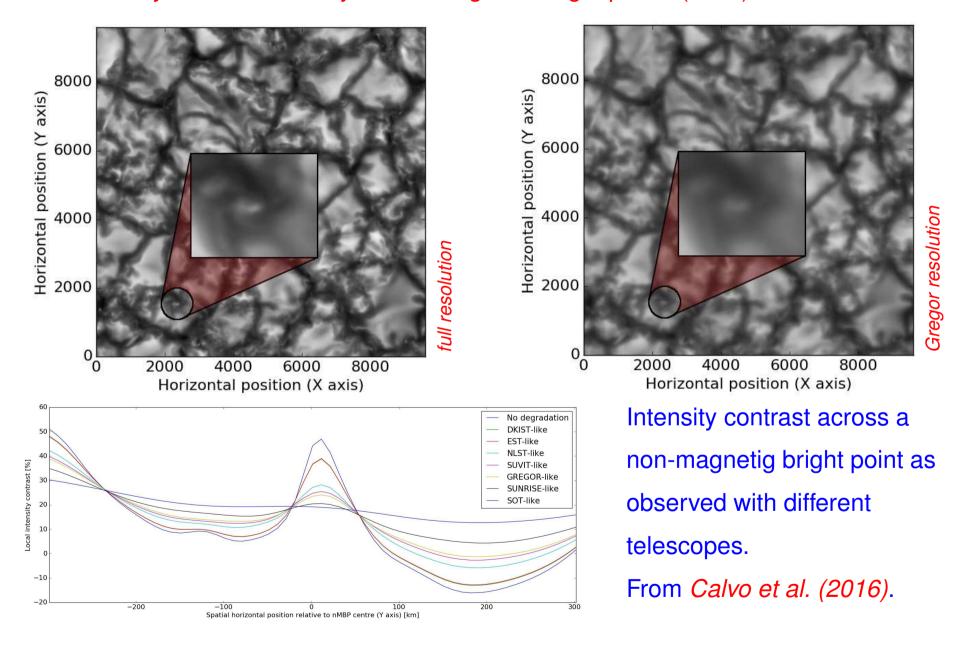
One can then estimate the magnitude of v_{θ} . With

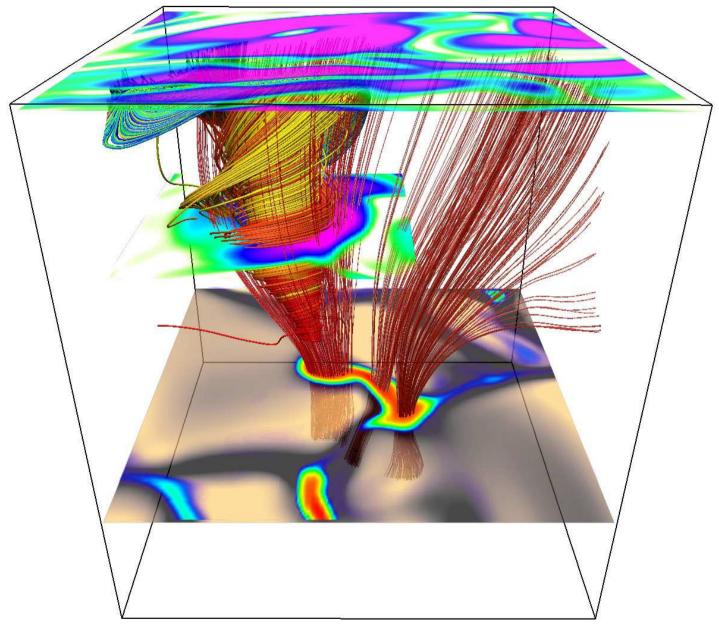
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one obtains with $C_{
ho} pprox -0.5$ and $C_{T} pprox 0$

$$v_{\theta} = \sqrt{R_{\rm s} \, T_{\rm eff} \, [1 - (1 + C_{\rho})(1 + C_T)]} \approx \sqrt{\frac{R_{\rm s} \, T_{\rm eff}}{2}} = 4.4 \, {\rm km \, s}^{-1} \, ,$$

while the maximum azimuthal velocities in the simulation are $v_{\theta}^{\rm max} \approx 6 \, {\rm km s}^{-1}$.



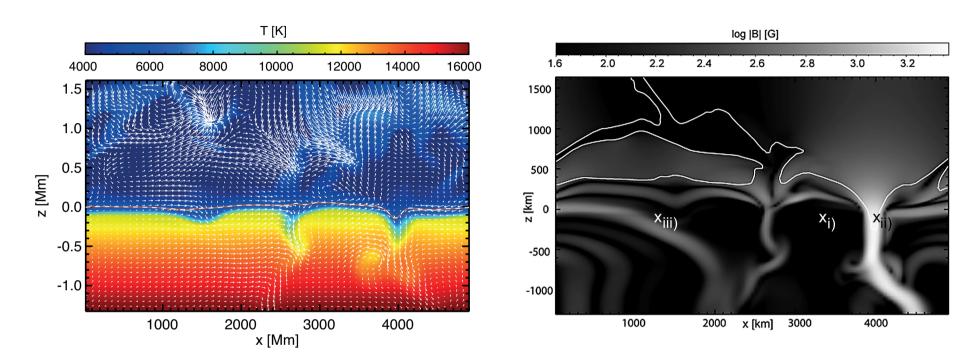


Close-up of a swirl event. The footpoints of the magnetic flux concentrations (red) are trapped in swirling downflows of the surface layers of the convection zone. In the outer atmosphere, the frozen-in plasma co-rotates with the magnetic field (spiral streamlines). From www.solartornado.info, Wedemeyer et al. (2012)

In this example, numerical simulations served to make a *prediction*. Because the simulations faithfully reproduce observed features like granules, the granular rms intensity contrast, or the shape of spectral lines, we can have some faith in the simulations. Therefore, new features, like swirling non-magnetic bright points, can, with some confidence, be predicted to really exist on the Sun. The complex structure of the non-magnetic bright points found in the simulations got *reduced to an analytical toy model*.

The next paragraph treats an example of *virtual experimentation*. Since we cannot take the Sun in the laboratory and since we cannot travel to the Sun and cary out experiments in situ, we reconstruct it in the computer for carrying out experiments.

§ 5 Case study III: Virtual experiments with numerical simulations

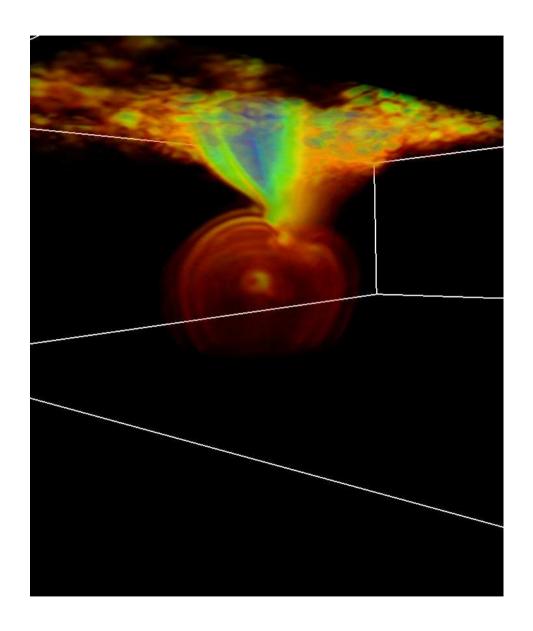


Temperature (colors), velocity (arrows), and optical depth $\tau_c=1$ (dashed curve).

Magnetic field strength (gray scales), level where $c_s = c_A$ (white contour), locations of local wave excitation (crosses).

Movies of wave excitation at \times_i , \times_{ii} , \times_{iii} , and along the lower boundary.

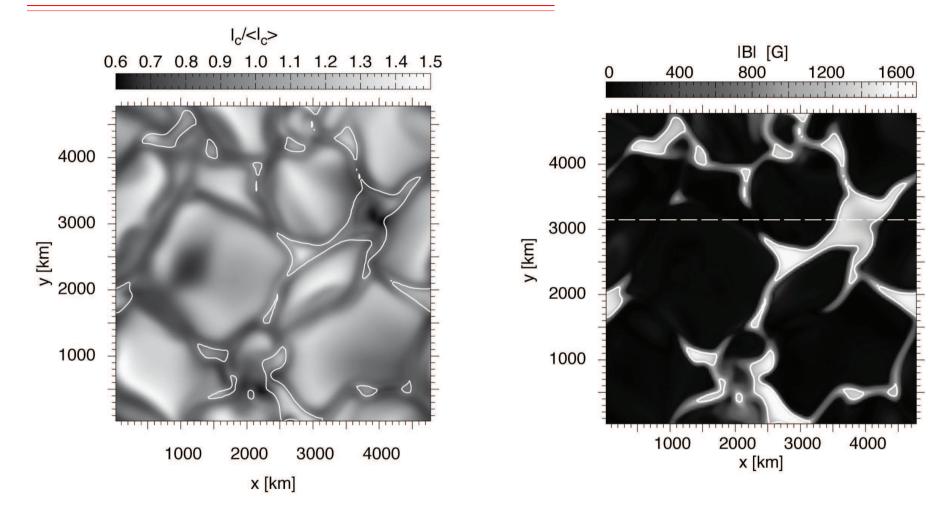
5. Case study III: Virtual experiments with numerical simulations (cont.)



Time instant of a spherical, fast acoustic wave, initiated by a local pressure perturbation in the convection zone. When the wave encounters the low beta magnetic flux concentration in the photosphere it partially converts into a fast magnetic mode, which shows the typical "faning out" already encountered in the 2-D simulation. Colors show absolute velocity perturbation.

Courtesy Christian Nutto, KIS.

§ 5.1 Magnetic halos and shadows

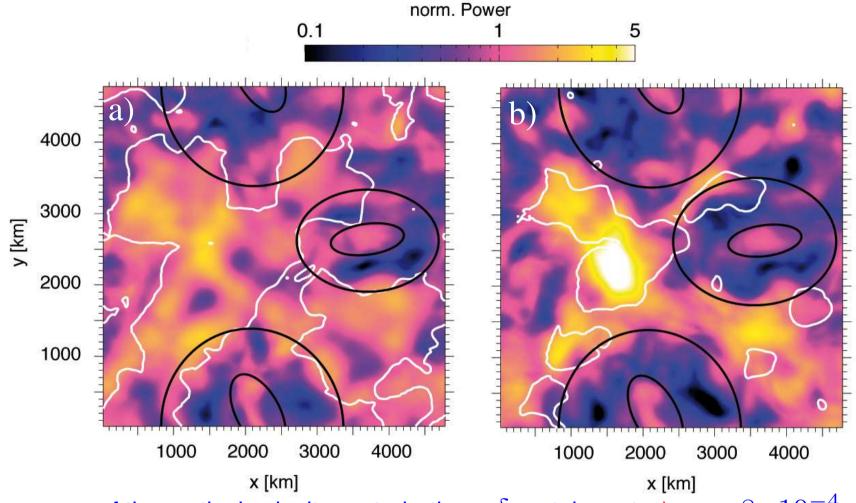


Left: FOV of $6.6'' \times 6.6''$ in white light. *Right:* Magnetic field strength at $\langle \tau_c \rangle = 1$.

Contours: Equipartition level where $c_s = c_A$. From Nutto et al. 2012, A&A 542, L30.

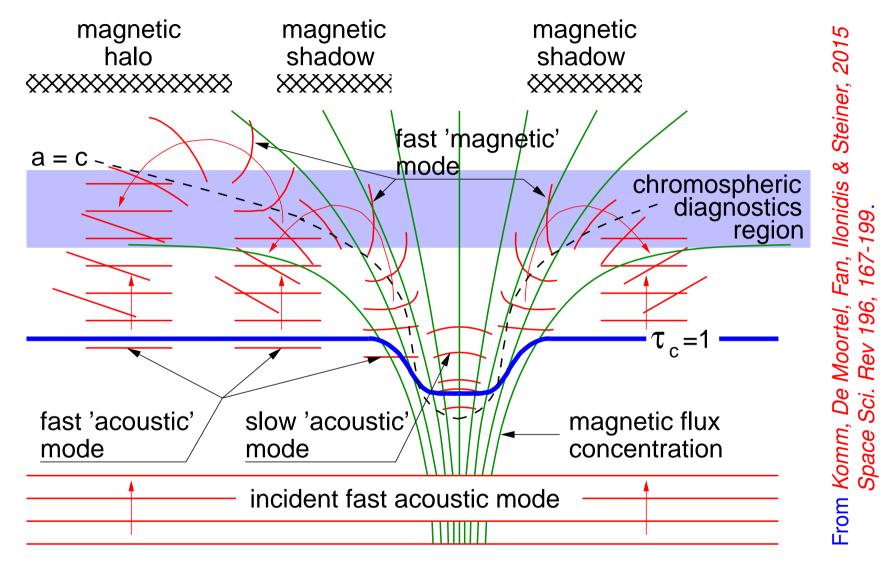
5.1. Magnetic halos and shadows (cont.)

toc — ref



Power maps of the vertical velocity perturbations, δv_z , taken at a) $\tau_c = 8 \cdot 10^{-4}$ and b) $\tau_c = 6.7 \cdot 10^{-5}$. The white contours shows the equipartition level $c_s = c_A$. The ellipses mark regions where the magnetic shadow can be identified. Note suppression of power in the region between the large and the small ellipses. From Nutto et al. 2012.

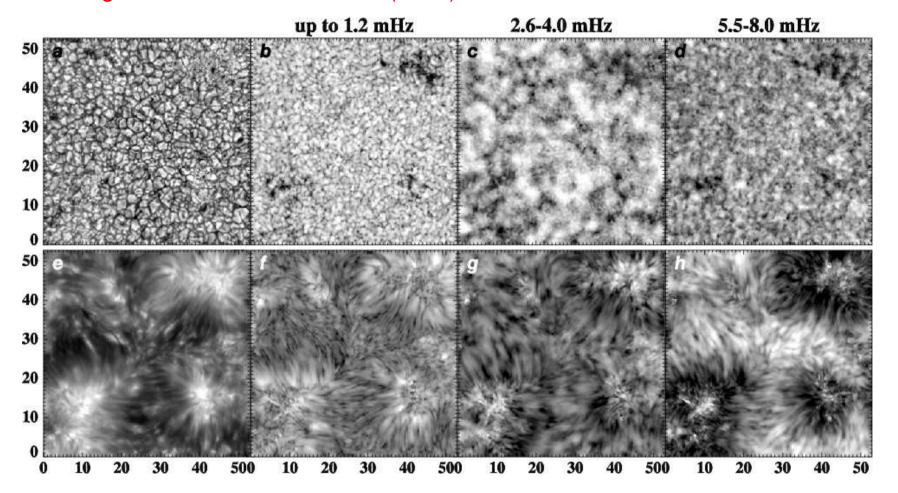
5.1. Magnetic halos and shadows (cont.)



Sketch of the three different magneto-acoustic modes that lead to the phenomenon of the magnetic shadow and the magnetic halo.

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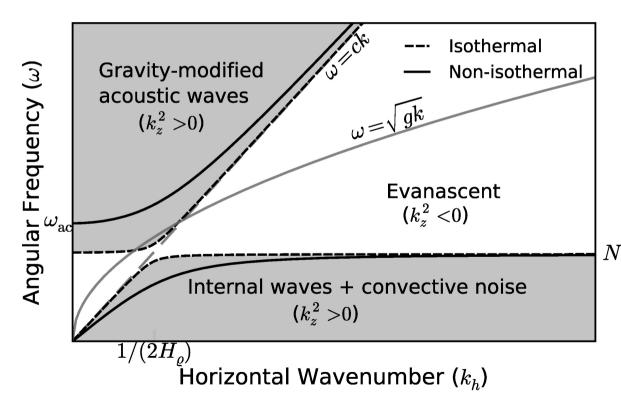
5.1. Magnetic halos and shadows (cont.)



a) Broadband continuum at 710 nm. e) Line core intensity of Call 854.2 nm. b)-d) and f)-h) Logarithm of the Fourier Doppler-velocity power averaged over the indicated range of frequencies of the photospheric line Fe I 709.0 nm (b)-d)) and the chromospheric line Ca II 854.2 nm (f)-h)). From Vecchio, Cauzzi, Reardon et al. (2007), A&A 461, L1. obtained with IBIS at DST.

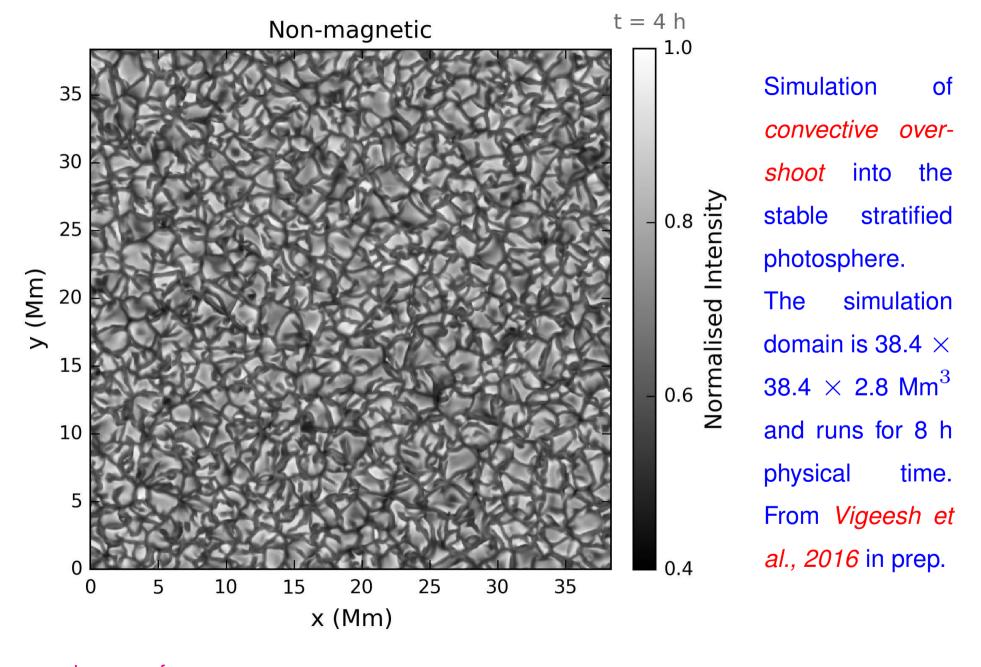
§ 5.2 Gravity waves

Gravity waves occur in the stable stratified (subadiabatic) part of the solar atmosphere, i.e., above the convection zone. Their restoring force is *buoyancy*. They are excited by *convective overshoot* into the photosphere.



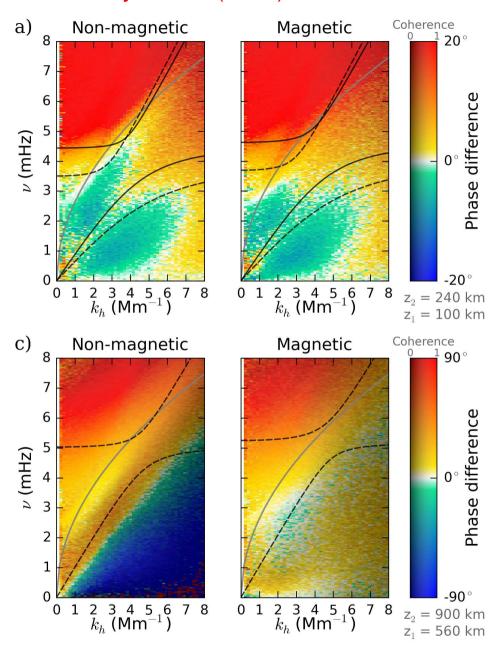
Schematic of the *diagnostic diagram* for a particular height in the atmosphere. $\omega_{\rm ac} = c_s/(2H_p)$ is the acoustic cutoff period, N the Brunt-Väisälä frequency. Internal, or gravity waves have frequencies $\omega < N$.

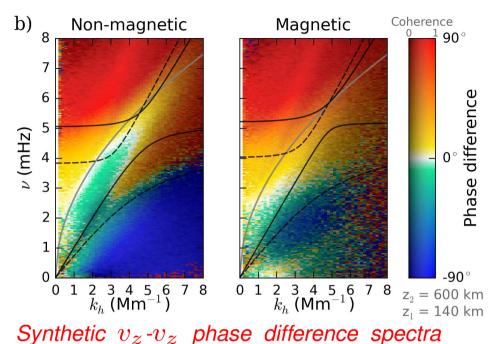
5.2. Gravity waves (cont.)



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5.2. Gravity waves (cont.)



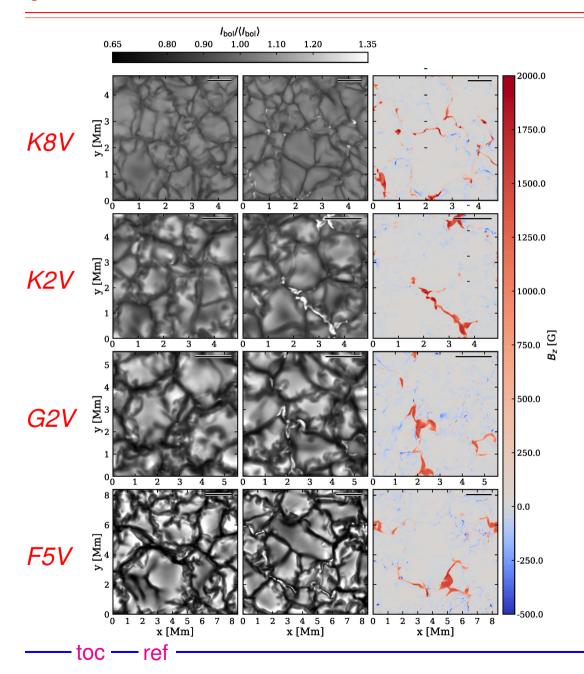


between heights of *a*) z = 100 km and z = 240 km, *b*) z = 140 km and z = 600 km, and *c*) z = 560 km and z = 900 km, for the *non-magnetic model* (left) and the *magnetic model* (right). Note the absence of of upwardly propagating gravity waves in the upper layers of the atmosphere of the mangnetic model. From *Vigeesh et al., 2016*.

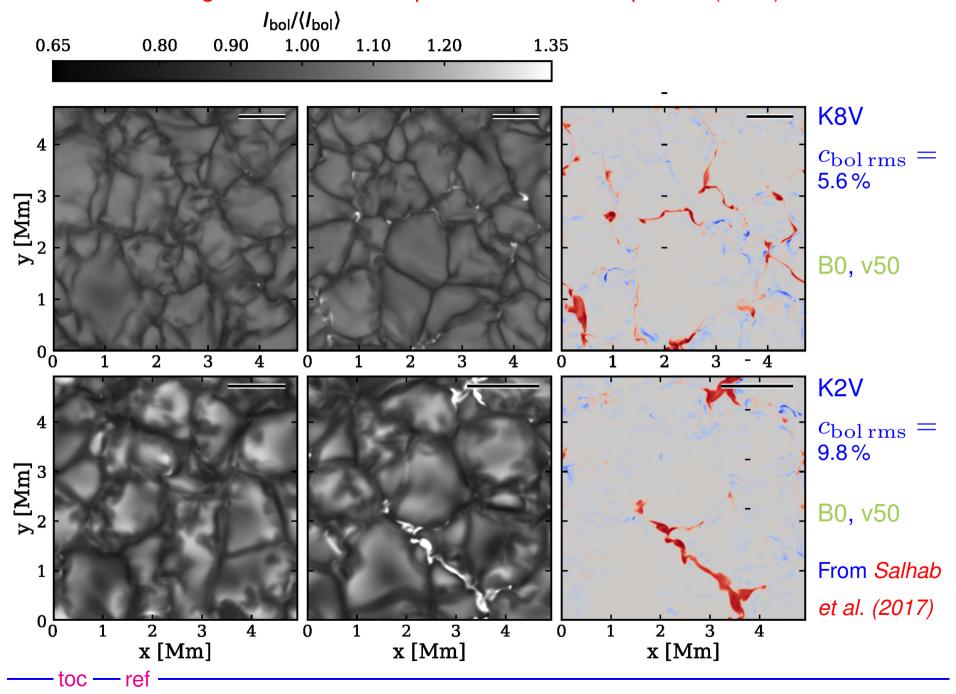
5.2. Gravity waves (cont.)

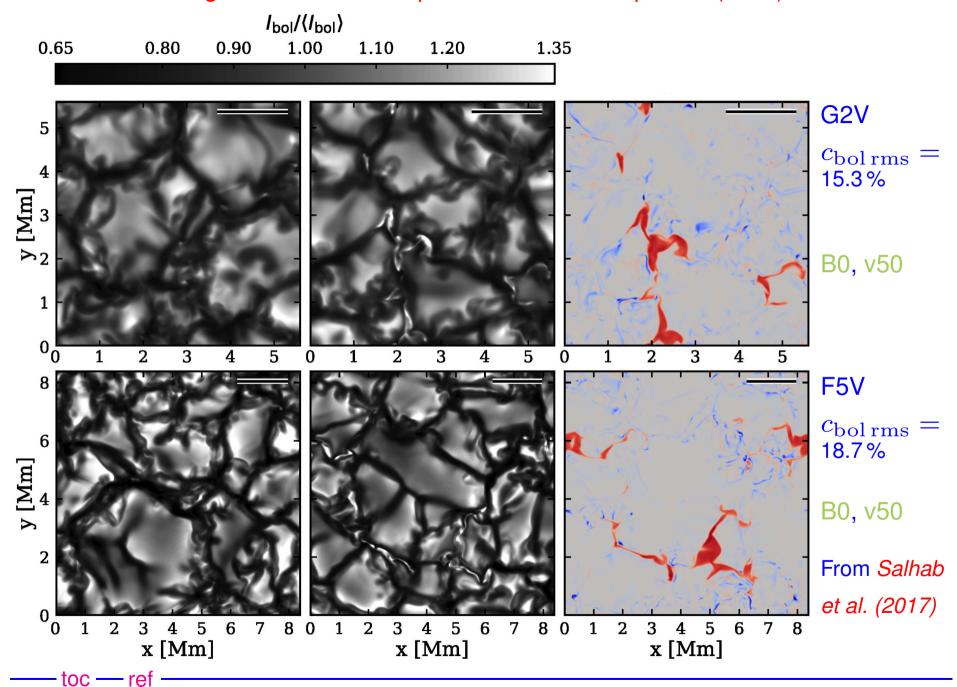
Possible explanations for the absence of propagating internal waves in the upper atmosphere of the magnetic simulation are:

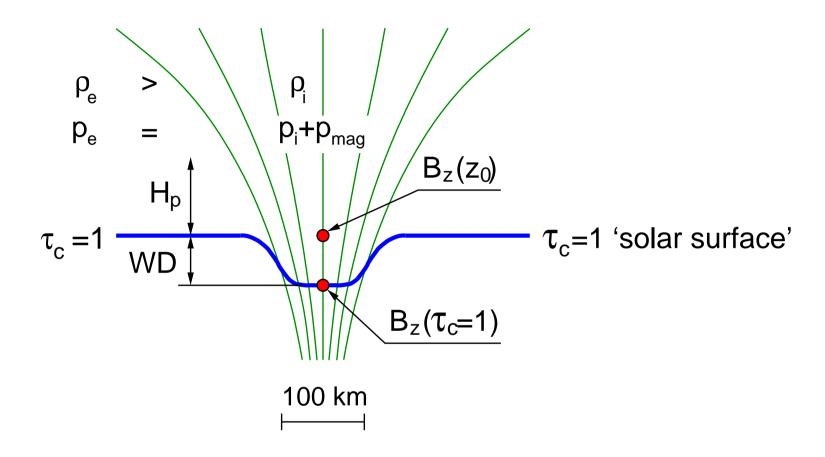
- *Mode conversion* of the internal wave *to Alfvén waves* (unlikely in the present case because to the predominat vertically directed magnetic field);
- Mode coupling to magneto-acoustic waves and *reflection* back into the atmosphere as described by Newington & Cally (2010, 2011).
- Non-linear interaction of internal waves with shear flows, leading to the breaking of internal waves into turbulence. (shear flows are provided by swirling motion in the upper atmosphere induced by the mangetic field)



- "Box in a star" simulations of the surface layers of four spectral types;
- Each simulation is run twice:
 with and without magnetic fields;
- Initial vertical homogeneous
 field of 50 G and 100 G;
- Multi-group radiation transfer using 5 opacity bins;
- Numerical, non-stationary, three-dimensional radiation magnetohydrodynamics using the CO⁵ BOLD code.







Magnetic flux concentration *(green)* with optical surface $\tau_c=1$ *(blue)*, and "Wilson depression" WD.

	spectral type	K8V	K2V	G2V	F5V
	initial B_z [G]	50	50	50	50
1	${\sf rms}(B_{z\ { m MBF}}(z_0))$ [G]	1438 ±40	1358 ±42	1282 ±40	1248 ±32
2	$\mathit{rms}(B_{z \; \mathrm{MBF}}(au_{\mathrm{R}}=1))$ [G]	1575 ±53	1598 ±93	1480 ±84	1565 ± 81
3	$\max(B_{z\mathrm{MBF}}(z_0))$ [G]	2204 ±79	1871 ±63	1739 ±73	1675 ±85
7	$rms(B_z(z_0))$ [G]	249.9 ±5.5	248.0 ±8.0	238.3 ±7.2	237.9 ±6.5
8	$p_{ m gas}(z_0)$ [kPa]	<i>26.9</i> ±0.1	15.2 ±0.1	10.5 ±0.2	7. <i>52</i> ±0.2
12	$B_{ m eq\;th}(z_0)$ [G]	2596 ±5	1951 ±7	1614±12	1362 ± 15
14	$B_{ m eq\ tot}(z_0)$ [G]	2681 ±5	2058 ± 7	1765 ± 11	1550 ± 13
15	$ ho_{ m int}/ ho_{ m ext}(z_0)$ [-]	0.75 ± 0.02	0.54 ± 0.03	0.46 ± 0.04	0.36 ± 0.05
16	$eta(z_0)$ [-]	2.7 ± 0.2	1.3 ±0.1	0.74 ± 0.1	0.38 ± 0.1

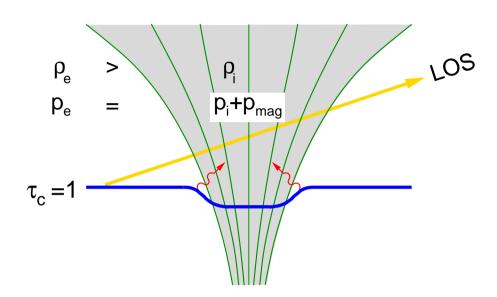
ullet Constant $B_z(au=1)pprox 1550\,\mathrm{G.}$ ullet Super-equipartition magnetic fields for F5V and partially for G2V. ullet Increasing evacuation with increasing T_{eff} .

—— toc — ref -

Conclusion: \bullet $B_{z\,\mathrm{MBF}}(\tau_\mathrm{R}=1)\approx 1550\,\mathrm{[G]}$, fairly independent of spectral. type \bullet Maximal field strengths can be superequipartition for F5 but are clearly subequipartition for K8 and K2. \bullet The evacuation monotonically increases with increasing effective temperature, T_eff .

Conclusions of Steiner, Salhab, Freytag et al. (2014), PASJ 66, S5

Magnetic flux sheath in a



 ρ_{e} > ρ_{i} ρ_{i} ρ_{c} = $\tau_{c} = 1$

K-type atmosphere

G-type atmosphere.

Small depression of the $au_c = 1$ surface

⇒ weak hot-wall effect.

Weak evacuation

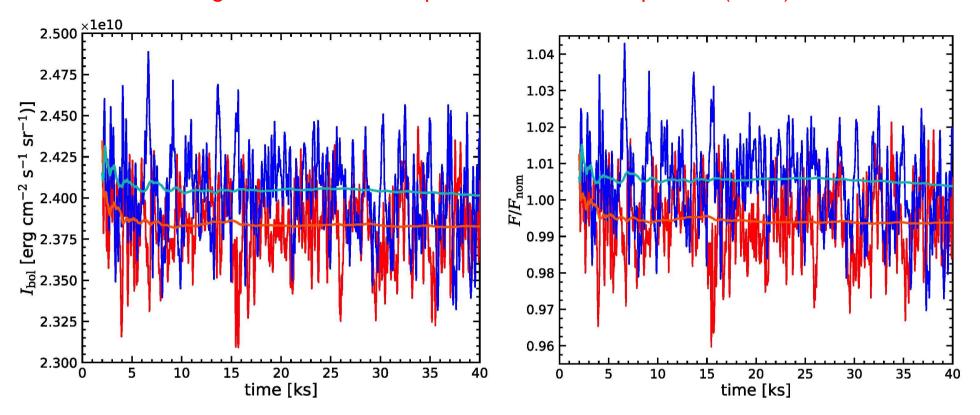
 \Rightarrow faint facular granules.

Large depression of the $au_c=1$ surface

⇒ strong hot-wall effect.

Strong evacuation

⇒ bright facular granules.



 $I_{\rm bol}(t)$ (left) and $F_{\rm bol}(t)$ (right) leaving the computational domain in the vertical direction through the top boundary for the *magnetic* (blue curve) and the *non-magnetic* (red curve) solar model (G2V). Cyan and orange curves are the expanding means.

$$I_{\mathrm{bol}}(t) = \langle I_{\mathrm{bol}}(\boldsymbol{\hat{z}},t) \rangle \; ; \qquad F_{\mathrm{bol}}(t) = \left\langle \int_{4\pi} I_{\mathrm{bol}}(\boldsymbol{n},t) \, \boldsymbol{n} \cdot \boldsymbol{\hat{z}} \, \mathrm{d}\Omega \right\rangle \, .$$

	spectral type	K8V	K2V	G2V	F5V
	initial B_z [G]	50	50	50	50
25	$\delta_{I_{ m bol}}$ [%]	0.25 ±0.2	0.68 ±0.9	0.88 ±1.1	0.53 ±0.8
26	$\delta_{F_{ m bol}}$ [%]	<i>0.39</i> ±0.2	<i>0.86</i> ±0.9	1.15 ±1.1	<i>0.95</i> ±0.8
27	$\delta_{F_{ m bol}} - \delta_{I_{ m bol}}$ [%]	0.14	0.18	0.27	0.42
30	WD_w [km]	60 ±14	139 ±34	232 ±65	388 ±113
31	$ extsf{WD}_{ ext{w}}/H_p(au_R=1)$ [-]	$0.7\pm$ 0.1	1.3 ±0.3	1.4 ±0.3	2.6 ±0.7
15	$ ho_{ m int}/ ho_{ m ext}(z_0)$ [-]	0.75 ± 0.02	0.54 ±0.03	0.46 ±0.04	0.36 ±0.05
16	$eta(z_0)$ [-]	2.7 ±0.2	1.3 ±0.1	0.74 ± 0.1	0.38 ±0.1

Radiative surplus of the magnetic over the field-free models, weighted mean Wilson depression, and degree of evacuation of the flux concentrations.

Conclusion: • For all spectral types considered here, the small-scale magnetic fields produce a *surplus in radiative intensity and flux*. It is most pronounced for G-type and early K-type stars.

ullet The difference $\delta_{F_{
m bol}} - \delta_{I_{
m bol}}$ is always positive and monotonically increases with increasing effective temperature, owing to the monotonically increasing Wilson depression and degree of evacuation.

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